

The Timing of International Financial Market Bubbles

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ABSTARCT

We employ a recursive econometric technique to identify and date multiple financial bubbles in five countries, the US, UK, France, Germany, and Japan. We identify multiple bubbles in each country except Germany for the period, 1973-March 2018. These bubbles are classified into three groups, each clustered around a significant financial event. The results show that if there is no synchronicity in the timing of bubbles, overall annualized returns are positive even after the bubbles burst. However, during the subprime mortgage crisis bubble, all nations experienced simultaneous, short bubbles with returns falling sharply in all countries. Thus, one can conclude that if bubbles do not coincide investors are better off riding out the bubble.

Key words: Financial bubbles, bubbles, financial crisis

JEL: M16, M21

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1. Introduction

This paper uses an econometric technique to determine the presence of stock price bubbles across stock markets in five different countries: the United States, France, Germany, the United Kingdom, and Japan. There are several advantages of identifying bubbles. First, if bubbles are detected early, then a portfolio manager can take precautions to exit the market before the bubble bursts. Second, if a bubble can be identified in real-time, the Central Banks of the affected countries may seek to lower asset prices to mitigate the adverse effects of a bubble.

Before identifying a bubble, we must first define what we mean by the term. Peter Garber's book *Famous First Bubbles* (2001) defines a bubble as:

The definition of bubble most often used in economic research is that part of asset price movement that is unexplainable based on what we call fundamentals. Fundamentals are a collection of variables that we believe should drive asset prices. In the context of a particular model of asset price determination, if we have a serious misforecast of asset prices we might then say that there is a bubble. (p.4).¹

In a series of papers, which include Phillips and Yu (2011), Phillips, Shi, and Yu (2015a), Phillips, Wu, and Yu (2011), and Phillips, Shi, and Yu (2015b), the authors develop an econometric technique to identify the timing of the beginning and the end of a financial bubble. Most simply put, the technique determines statistically when assets are selling at a greater price than is justified by the fundamentals. The bubble ends when prices return to levels justified by fundamentals.² The technique can detect bubbles in real time, requiring only data available up to the beginning of a financial bubble to make the identification, as well as identify multiple bubbles across a longer time series. This technique offers significant advantages over earlier methods, such as cointegration, which only works retrospectively. Phillips, Shi, and Yu (2015a) applied this procedure to identify the beginning and ending of multiple bubbles in S&P 500 and NASDAQ indices. In their model, the index dividends are used as the fundamentals that justify the price level. In the literature, dividends are viewed as predictors of stock prices and this belief is derived from the present value theory (see Shiller (2015) and Cochrane (2001, 2005)).

The first objective of this paper is to identify the presence of multiple stock price bubbles in the United States, United Kingdom, Japan, Germany and France.³ The second objective is to date the beginning and ending of bubbles in each of the above five stock markets. Although, the focus is not on the correlation of timing, we also investigate if stock price bubbles coincide. Should bubbles coincide, one plausible explanation could be that a bubble in one market leads to bubbles in other stock markets due to the integration of capital markets. On the other hand, if bubbles do not coincide, a bubble in one country may drive funds into that country to take advantage of the rising stock prices resulting from the bubble. In turn, this inflow of funds strengthens the original bubble, while also lowering returns in other countries, making bubbles in these other countries less likely. To begin to investigate this relationship we need to first examine the empirical evidence.

¹ Garber's definition of a bubble follows a long literature, including, but not limited to, Mackay (1841), Kindleberger (1978) and Shiller (2015).

² Eugene Fama, the Nobel Prize winning champion of the Efficient Market Theory, claims there is no systematic way of identifying the existence of a financial bubble. In his 2014 Nobel address he argues bubble proponents seem to define it as a substantial price increase, unjustified by economic fundamentals, which is followed by a sharp and anticipated price decline. Robin Greenwood, Andrei Shleifer, and Yang You (2017), operationalize this definition in their study of industries by specifying both a minimum upward return and minimum subsequent downturn during a specific time period.

³ These five stock markets capture 54.4% of the world market capitalization.

We find evidence of multiple bubbles in four of these five countries. We also find that timing and magnitude of the bubble vary from country to country and that most of the financial bubbles we observe do not spread from the market of one country to another. However, in the case of exceptionally severe financial bubbles, such as the 2007/2008 housing market bubble that began in the United States, we find a strong connection. However, even in cases when financial bubbles are connected, we observe significant variation in their start and end dates.

In the next section, we briefly outline the econometric technique employed to identify financial bubbles. Caspi (2014) provides a more detailed explanation of the technique.

2. Methodology

It is well known that stock prices exhibit non-stationary behavior. The standard testing procedure for non-stationarity or unit roots is the Augmented Dickey Fuller (ADF) test.

$$P_t = \mu + \delta P_{t-1} + \sum_{i=1}^l \varphi_i \Delta P_{t-i} + e_t \quad (1)$$

where P_t is the stock price, μ is the intercept, and e_t , is random error term. The δ term is the key coefficient of the regression. In the usual unit root tests, the null hypothesis is that there is unit root ($\delta = 1$) and the alternative is the absence ($\delta < 1$) of unit root. The discussions around the explosive price bubbles research suggest testing the presence of unit root against the alternative of explosive autoregressive coefficient ($\delta > 1$). Phillips et al. (2011) develop one such test. Phillips et. al. (2011) suggest performing a series of unit tests using various subsamples of the data. While this test provides greater power than extant tests, such as cointegration, it was not well suited to identifying multiple bubbles over time. Thus, Phillips et al. (2015a, 2015b) generalized the technique to better detect multiple bubbles and improve the power of the tests. Their testing of bubbles is built on the methodology that current asset prices reflect discounted future dividends.

The tests in these papers are based on the reduced form equation as stated in equation 1, but with prices as defined by Campbell and Shiller (1989):

$$p_t = \mu + \delta p_{t-1} + \sum_{i=1}^l \varphi_i \Delta p_{t-i} + e_t \quad (2)$$

where p_t is now the price to dividend ratio of a countries stock index, μ is the intercept, and e_t , is random error term. The δ term is the key coefficient of the regression. For Phillips et al. (2011m, 2015a, and 2015b), the null hypothesis postulates the presence of a unit root and the alternative hypothesis claims the data contains a mildly explosive autoregressive coefficient. A mildly explosive autoregressive coefficient implies prices are veering uncontrollably away from the fundamental value, as represented by the price-dividend ratio. As such, the data is consistent with the existence of a bubble as the equity price greatly exceeds the level justified by the fundamentals as represented by the dividend level. Formally, we may state the null and alternative hypothesis as:

$$H_0: \delta = 1 \quad H_1: \delta > 1$$

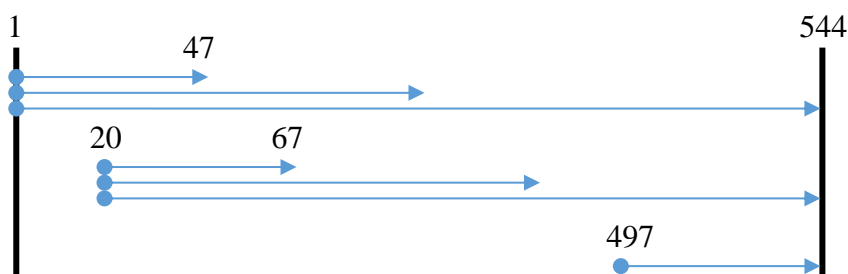
We shall couch our explanation of the test procedures in terms of our data. The identification of multiple bubbles characterized by intermittent collapsing behavior requires higher frequency data, and our dataset consists of 544 monthly observations of the stock price to dividend ratio in five countries (the US, UK, France, Germany and Japan) covering the period of January 1973 thru March 2018. These data are taken from *Thomson Reuters Datastream*. We begin with a description of the supremum augmented Dickey-Fuller (SADF) test, which is used to test for the overall existence a single bubble. We then move to the generalized supremum augmented Dickey-Fuller (GSADF) test, which is used to test for multiple bubbles. Finally, we discuss the backwards

supremum augmented Dickey-Fuller (BSADF) test which is used for identifying the beginning and end of multiple bubbles.

For the supremum augmented Dickey-Fuller (SADF) test, we begin by sampling a window of the first 47 months⁴ of data and the ADF for this interval is calculated. Next, we extend the window by adding one month (thus covering months 1-48) and an ADF is again computed. This continues until the window encompasses the entire sample (months 1-544). Thus, 497 ADF statistics are computed. The supremum (highest) of these ADF statistics is the SADF. To identify the beginning and the ending of a bubble, these recursive ADF statistics are compared against the right-tailed critical values of the asymptotic distribution of the standard Dickey-Fuller test-statistics. Phillips et al. (2011) use simulation methods based on the distribution of the ADF statistics to derive the critical statistics for the SADF statistic for each sample. A bubble occurs when the SADF statistic exceeds the critical value. While the SADF test is useful for identifying the overall presence of a bubble, it is not well suited for determining the presence of more than one bubble in the data.

Phillips et al. (2015a, 2015b) show that when the sample period includes multiple bubbles, the SADF test may suffer from reduced power and may fail to detect correctly these bubbles. To overcome this weakness, they suggest a generalized SADF test (GSADF). While the SADF test takes the supremum ADF statistic over windows that begin at the first observation and expand to the last observation, the GSADF also allows the starting point of these windows to vary. For example, one recursion may start at period 20 with an initial window that goes through period 67. Like the SADF calculation, one period is then added, and the regression is rerun. This continues until the window contains the entire remaining sample. Then a new starting point is chosen and the procedure is repeated. This continues until the all potential starting points have been exhausted, effectively calculating ADF statistics for every possible window of at least 47 contiguous months within the data. Figure 1 below illustrates this procedure.

Figure 1 - Process of Performing a Recursive Regression for the GSADF test



The GSADF is then the largest ADF statistic across all feasible windows (each recursion). Again, Phillips et al. (2015a, 2015b) determine critical GSADF statistics by employing simulation. If the values of the GSADF for the whole period exceeds the critical value, we find the data consistent with the presence of a bubble. While this approach provides a more robust identification of one or more bubbles, it does not date the bubble(s).

Using the GSADF test to date bubbles in real time requires performing a backward extending windows from the current point, which Phillips et al. (2015a, 2015b) labelled as a backward SADF (BSADF) test. This process is very similar, but in reverse, to the GSADF process described above.

⁴ The minimum window length r_0 was set using $r_0 = 0.01 + 1.8\sqrt{T}$ where T is equal to the number of observations. This results in a minimum window length of 47 months or about 8.7% of the observations.

For example, to test for a bubble at month 67, the first ADF statistic is calculated from observations 20 to 67 (or going back 47 months from month 67). The window is then expanded backwards one observation (now covering months 19 to 67) and another ADF statistic is calculated. This process of expanding the window continues until the first observation is reached. The BSADF statistic for the starting date is then the supremum (highest) value across these backward expanding windows. A sequence of BSADF statistics generated by repeating this process for all possible starting points. When this BSADF sequence crosses a critical value from below a bubble begins and when it crosses from above the bubble terminates. Figure 2 below illustrates this procedure.

Figure 2 - Process of Performing a Backwards SADF test



3. Results

We consider a historical time series for the five countries: The United States, the United Kingdom, France, Germany, and Japan. The data comprise the stock price index and the index dividends in each of the five countries for the major market indices. For the US, the S&P 500 index and the monthly dividends are used to compute the price-dividend ratio (p-d-ratio). Equivalent measures of stock market indices and dividend yields for the UK, France, Germany and Japan are obtained from *Datastream*. The data are monthly observations between January 1973 and March 2018 for each country. The computed price-dividend ratio at any given time reflects how stock prices relate to their fundamentals at that time. If there is no bubble component, then the series will either be stationary or contain a unit root. As Cochrane (1992) and Ang and Bekaert (2007) show, tests for a unit root in this series do not eliminate the possibility of a non-stationary discount factor influencing the ratio. In addition, Phillips et al. (2015b) argues these types of tests are unduly conservative.

We first apply the GSADF test to the price-dividend ratio. Table 1 presents the critical values for the test and the test statistic for each country. In all countries except in Germany, we reject the null hypothesis of unit roots in favor of the explosive right tail alternative. The critical values are obtained from 1,000 Monte Carlo simulations with a random walk data generating process. As is evident from Table 1, the GSADF statistic exceeds the 5% critical values in all countries except Germany. This provides strong evidence that the stock price-dividend ratio had explosive sub-periods in four countries. We fail to reject the null hypothesis for Germany that the stock price-dividend ratio contains a unit root, and for the rest of our analysis we restrict our attention to the remaining four countries. We conclude from the GSADF test that there is evidence of one or more bubbles in these remaining four countries.

Table 1 - GSADF Test Results by Country

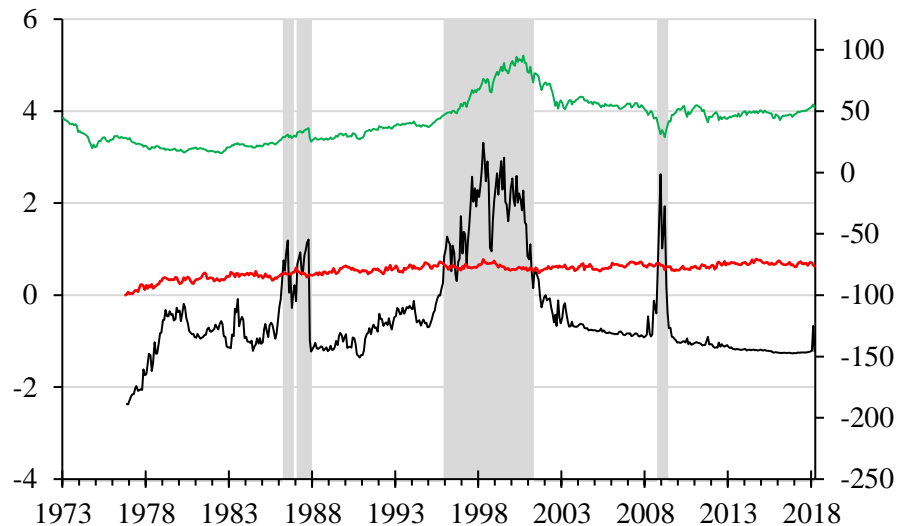
	t-Statistic	Probability
United States	3.315	0.002
France	3.104	0.003
United Kingdom	2.365	0.035
Japan	6.368	<0.0000
Germany	2.025	0.099

Right tailed ADF with critical values of 90% at 2.0196, 95% at 2.250 and 99% at 2.8899

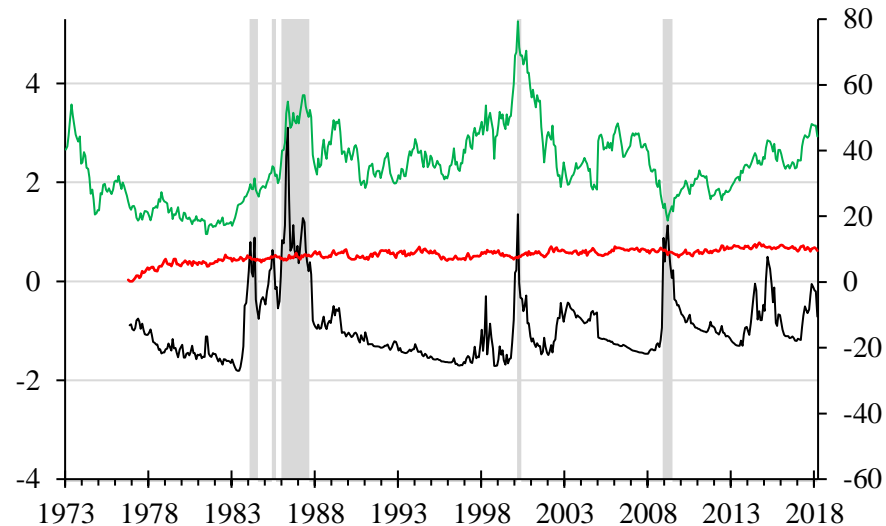
Next, we apply the BSADF test to conduct a real-time bubble detection exercise for the remaining four countries to identify and date market bubbles. The results are shown in Figure 3 in which the black lowest line represents the BSADF statistic of each country on each date. The red middle line represents the 95% critical values for each BSADF statistic. The green top line represents the price-dividend ratio for each country. A bubble begins when the BSADF statistic (black line) first exceeds the critical value (red line). We define the end of the bubble to be when the BSADF drops below the critical value for two consecutive months. The figure clearly shows that each of the four countries experienced some months of explosive and non-explosive periods.

Table 2 presents the beginning and end dates for which bubbles were identified using the BSADF approach in each country, the bubble duration, the index price at the start and end of each bubble, and the annualized rate of return during the bubble period for the index. Four bubbles were detected for the US, five for France, four for the UK and six for Japan. However, some of these bubbles are in very close proximity and may reflect related market adjustments. Figure 4 presents the timing of bubbles in alternate way, to more clearly show timing relative to the other countries. For ease of discussion, we will discuss these bubbles in three groups, each clustered around a significant financial event. Group 1 contains bubbles that occurred leading up to and after the global Black Monday stock market crash that occurred in October 1987. Group two contains bubbles that are clustered around the U.S. Dot-com Bubble which occurred roughly from 1995 to 2000. Group three contains bubbles that occurred during the 2008-2009 subprime mortgage crisis.

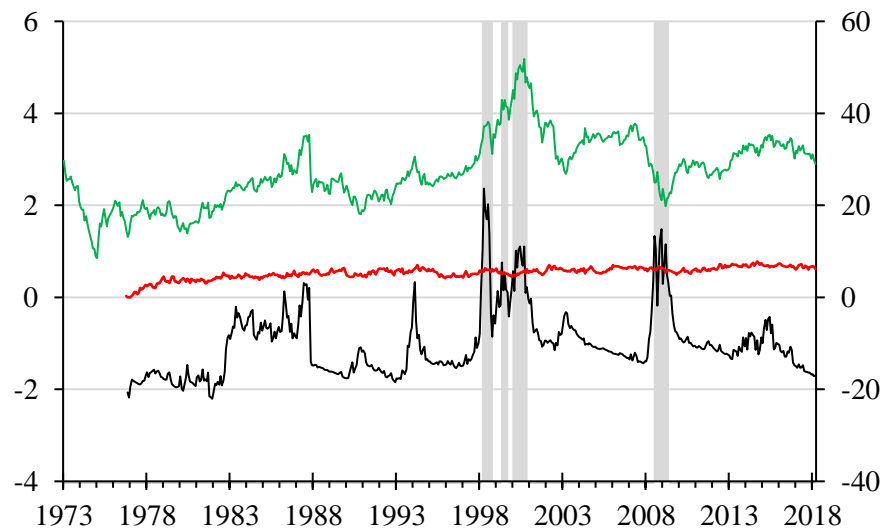
Figure 3 - Estimation Results by Country



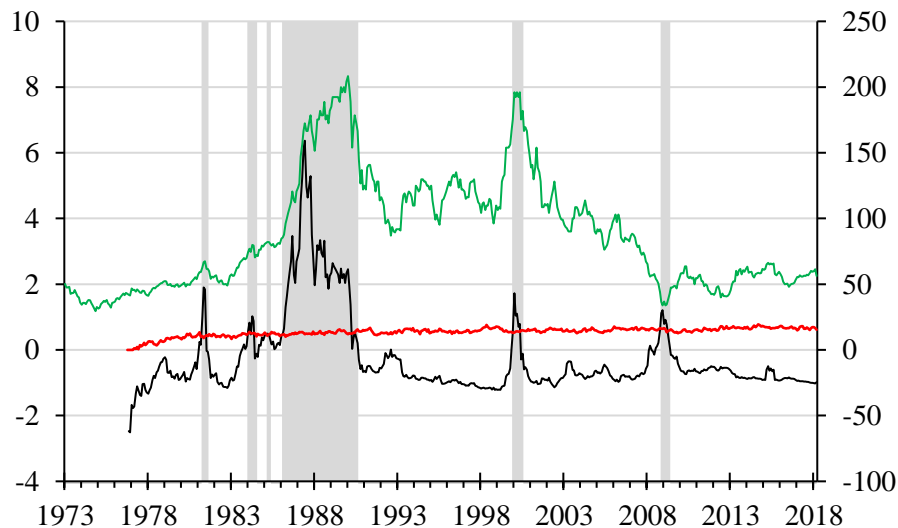
(a) United States



(b) France



(c) United Kingdom

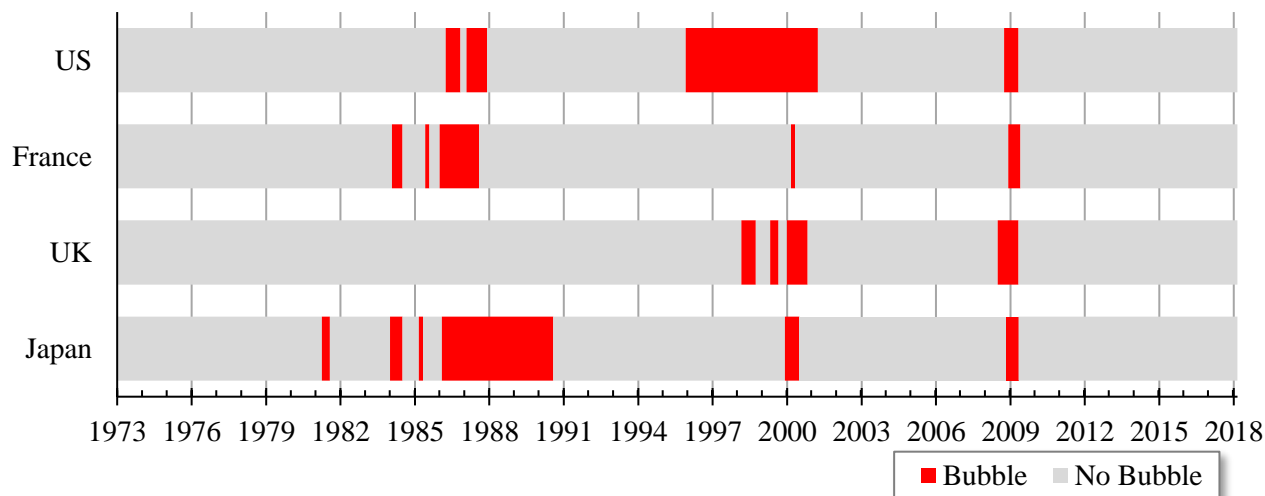


(d) Japan

— Backwards SADF sequence (left axis)
— 95% critical value sequence (left axis)
— Price-dividend ratio (right axis)

Table 2 – Financial Bubbles and Annualized Returns by Country

	Bubble Start	Bubble End	Duration (Months)	Index Start Price	Index End Price	Annualized Return ⁵
United States						
<i>Bubble 1</i>	1986M04	1986M11	8	180.16	200.63	15.4%
<i>Bubble 2</i>	1987M02	1987M12	11	195.09	193.55	-0.8%
<i>Bubble 3</i>	1995M12	2001M04	65	529.80	1306.07	17.8%
<i>Bubble 4</i>	2008M10	2009M05	8	1329.63	985.60	-32.9%
France						
<i>Bubble 1</i>	1984M02	1984M07	6	130.73	125.21	-7.1%
<i>Bubble 2</i>	1985M06	1985M08	3	141.12	148.77	17.2%
<i>Bubble 3</i>	1986M01	1987M08	20	158.15	253.49	30.9%
<i>Bubble 4</i>	2000M03	2000M05	3	1406.45	1489.18	18.7%
<i>Bubble 5</i>	2008M12	2009M06	7	1005.82	936.07	-10.2%
United Kingdom						
<i>Bubble 1</i>	1998M03	1998M10	8	943.20	1060.19	16.9%
<i>Bubble 2</i>	1999M05	1999M09	5	1258.16	1249.30	-1.4%
<i>Bubble 3</i>	2000M01	2000M11	11	1389.73	1347.51	-3.0%
<i>Bubble 4</i>	2008M07	2009M05	11	1438.74	985.60	-31.5%
Japan						
<i>Bubble 1</i>	1981M04	1981M08	5	98.41	91.61	-13.3%
<i>Bubble 2</i>	1984M01	1984M07	7	131.47	125.21	-7.1%
<i>Bubble 3</i>	1985M03	1985M05	3	142.51	150.12	16.9%
<i>Bubble 4</i>	1986M02	1990M08	55	166.38	254.89	9.6%
<i>Bubble 5</i>	1999M12	2000M07	8	1317.33	1456.11	14.3%
<i>Bubble 6</i>	2008M11	2009M05	7	1159.17	985.60	-21.6%

Figure 4 – Timing of Financial Bubbles by Country

⁵ Annualized returns are based on the difference in index values from one month prior to the beginning of the bubble to the month after the bubble ends, in order to capture the full bubble. For example, the annualized return for the first bubble in the U.S. is based on the change in index value between 1986M03 and 1986M12.

3.1. Black Monday (1981-1990)

The first group of bubbles we focus our attention on are those leading up to, and following, the global stock market crash on Monday, October 19, 1987. The first bubble that might be considered part of this group was experienced by Japan alone, as early as 1981 when from 1981M04 to 1981M08 the Japanese market fell at an annualized rate of 13.3%. However, it is unclear if this brief bubble is related to those that followed, as none of the other countries experienced a bubble until several years after. It is more likely that the beginning of the bubbles in this group was the one, also experienced by Japan, between 1984M01 and 1984M07 with a -7.1% annualized return. This bubble was closely followed by a second, very brief, bubble from 1985M03 to 1985M05 (with a -7.1% annualized return) and a third, lengthy, 55-month bubble, over which of which the market rose by 9.5% per year on average. This long bubble in Japan was fueled in part by a Japanese corporate invention called “zaitech” wherein speculation became an integral part of corporate earnings statements. If we exclude the first bubble in 1981, then during the 80 months between 1984M04 and 1990M08 the Japanese market was in a bubble for 65 months, or over 80% of that time. Over this 80-month period, the total market return was a 9.6% per year on average. While the bubbles in Japan for this period began the earliest and lasted the longest among the four countries, both France and the U.S. also experienced bubbles.

During the time of the bubbles in Japan, three bubbles occurred in France. The first was from 1984M02 to 1984M07 when the French market fell by 7.1%. This bubble was followed first by a brief 3-month bubble from 1985M06 to 1985M08, during which the French market rose an annualized 17.2%, and a longer 20-month bubble from 1986M01 to 1987M08, which saw an increase in the French market of 30.9% per year on average. During both these later bubbles, the French market experienced strong growth and across all three French bubbles (from 1984M02 to 1987M08) the French market for at an annualized rate of 18.9%.

Finally, the United States experienced two bubbles during this time that occurred shortly before and after Black Monday itself. The first lasted for eight months (1986M04-1986M11), with an annualized return of 15.4% while the second lasted for eleven months (1987M02-1987M12), with a 0.8% decline in prices. These two bubbles encompass a period of 21 months (the 19 months prior to Black Monday and the two months following) over the entirety of which with the annualized market return was 3.6%.

While these bubbles occurred roughly at the same time, it is hard to see strong patterns in the timing of these bubbles. Based on the estimated start dates of these bubbles alone, there is some suggestion of bubbles appearing first in Japan then moving to France and finally to the US, which, coincidentally, mirrors the collapse of the stock market on the day of Black Friday itself. However, these early bubbles in Japan are likely the result of the “zaitech” speculation discussed above. In addition, based on the estimated BSADF values, the UK did not experience a bubble, despite being one of the countries in which stocks fell the sharpest. During some of the individual bubbles within this group market returns were negative (such as Bubble 1 and 2 in Japan, Bubble 1 in France and Bubble 2 in the U.S.). However, when we group those individual bubbles that are near each other together in each country, the overall market return during the general Black Monday grouping of bubbles is positive in all three countries, which is contrary to Fama’s (2014) understanding of a bubble.

3.2. U.S. Dot-com Bubble (1995-2001)

The most significant bubble in this group occurred in the US, in which the Dot-com bubble originated. This bubble lasted 65 months from 1995M12 to 2001M04, during which the US market

rose at an annualized rate of 17.8%. During this time, three bubbles occurred in the UK, one from 1998M03 to 1998M10 with an annualized return of 16.9%, one from 1995M05 to 1999M09 with an annualized return of -1.4% and one from 2000M01 to 2000M11 with an annualized return of -3.0%. The total return across all three of these bubbles was an annualized 12.6%. Both Japan and France also experienced brief bubbles. In Japan during an 8-month bubble between 1999M 12 and 2000M 07, the market rose at an annualized rate of 14.3% while in France during a 3 month bubble between 2000M03 and 2000M05 the market rose at an annualized rate of 18.7%.

As in the group of bubbles discussed in the previous section for Black Monday, here there is also some suggestion of bubbles spreading between countries. The earliest and longest bubble occurs in the U.S., due to this being the origin of the Dot-com bubble. From there, bubbles next occur in the UK followed later by brief bubbles in Japan and France. However, this is again far from conclusive. While bubbles do occur in all four countries during this period, those in France and Japan are extremely brief, which is consistent with the results of Hon et al. (2007) and Kohn Et al. (2017) who did not find widespread contagion in equity markets during the Dot-Com bubble. As with the bubbles grouped around Black Monday, the overall market return during the general grouping of bubbles during this Dot-com bubble period is positive in all four countries, which is again contrary to Fama's (2014) understanding of a bubble.

3.3. Housing Market Crisis (2008-2009)

The third and final group of bubbles are clustered around the 2007-2008 U.S. Housing Market Crisis and subsequent financial crisis. Here all the BSADF technique detects almost simultaneous bubbles in all four countries, which are all characterized by sharply declining market prices. In the US there is an 8-month bubble from 2008M10 to 2009M05, in the UK an 11-month bubble from 2008M07 to 2009M05, in Japan a 7-month bubble from 2008M11 to 2009M05 and in France a 7-month bubble from 2008M12 to 2009M06. Unlike the earlier groups of bubbles, these bubbles exhibit a strong contagion effect, as the US, UK, Japan and France all experienced deep losses of -32.9%, -31.5%, -21.6% and -10.2%. During this crisis, the bubbles occurred contemporaneously in all countries which most closely approximates the behavior described by Fama (2014) wherein sharp increase in stock prices, without a corresponding increase in fundamentals, leads to subsequent deep losses.

4. Summary and Conclusion

Our results show that the BSADF test identifies multiple stock price bubbles between 1973 and 2018 in four of the five countries we study. These bubbles can be readily grouped and related to three significant financial events: Black Monday, the Dot-Com bubble and the subprime mortgage crisis. For the first two groups of bubbles, those surrounding the Black Monday and Dot-Com bubble, there was little synchronicity. Bubbles occurred at different times in different countries, and for some countries no bubble occurred (such as for the UK in the first group) or at most a single brief bubble occurred (such as for France and to a lesser extent Japan in the second group). The time period spanning these two bubble groups was long (nine years across the first grouping and six years across the second) and returns were overall positive across all countries for the full duration. This suggests that when a bubble is long, and not directly tied to bubbles in other countries, there is time for the fundamentals to improve. In this case, the bubble eventually bursts by these fundamentals catching up to prices, rather than prices dropping to reflect lower fundamentals.

In contrast, in the third group of bubbles, those during the subprime mortgage crisis, all countries experienced simultaneous, short bubbles with returns falling sharply in all four countries. In this case, there is not enough time for the fundamentals to improve enough to justify the bubble and a true bursting of the bubble occurs with prices falling to match these fundamentals. Thus, if bubbles do not coincide across nations, investors are better off riding out the bubble.

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